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Social Infrastructure and the Preservation of Physical Capital: Equilibria and Transitional Dynamics

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Social Infrastructure and the Preservation of Physical Capital: Equilibria and Transitional Dynamics

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Abstract

We study the mechanisms according to which social infrastructure influences the preservation of physical capital and, consequently, economic growth. The model considers that social infrastructure is a specific type of human capital, which acts in order to preserve already existing physical capital, by, e.g., reducing the incentive for rent seeking or corruption. Using an innovative methodology in economics, the Gröbner bases, we study the equilibrium of our model and conclude for the existence of two feasible steady-states or of unicity according to different combinations of parameters, highlighting a trade-off between consumption and production on one hand and social infrastructure and physical capital accumulation, on the other. We also present sufficient conditions for saddle-path stability. Finally, we describe transitional dynamics and calculate welfare effects from which we show that strengthening social infrastructure increases welfare.

JEL Classification: C02, C62, O41, O43.

Keywords: Social Infrastructure, Physical Capital Depreciation, Endogenous Growth, Equilibrium Multiplicity, Gröbner Bases.

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1 Introduction

We explore the effects of social infrastructure on the preservation of physical capital and, consequently, on economic growth. This is an unexplored link in the theory of economic growth, even within the literature that relates institutions to growth. In fact, social infrastructure can be associated with the existence of institutions, formal and/or informal in nature, that may help to decrease corruption, rent seeking, and cheating while improving transparency and trust in the economic environment of a country, facilitating the preservation of the existing physical capital stock, and enhancing economic growth.

The role of institutions on the economic performance of countries became so important that it gave rise to a new branch in economics, designated by "institutional economics", which was born with the seminal work of North (1990), among others. Empirical work has emphasized the important contribution of good institutions to economic growth and development, and there is an important consensus on this conclusion, as we can see in the work of Hall and Jones (1999), Acemoglu *et al.* (2001, 2002), Easterly and Levine (2003), Dollar and Kray (2003), and Rodrik *et al.* (2004). In this study we follow this consensual view and assume that good institutions contribute to economic growth. However, we go further and consider that the channel is through the protection of physical capital or investment. In fact, empirical literature has found a negative relation between corruption levels and capital accumulation (Campos and Lien, 1999), corruption and productivity (Salinas-Jiménez and Salinas-Jiménez, 2007), social barriers and capital accumulation (Grafton *et al.*, 2007), social capital and corruption (Bjørnskov, 2003), a positive relationship between governance institutions and investment (Aysan *et al.,* 2007), responsibility and capital accumulation (Breuer and McDermott, 2009), and trust and capital accumulation (Yamamura and Inyong, 2010). Bu (2006) presented evidence according to which depreciation rates are higher in developing countries than in developed ones. According to the author and references therein, some of the explanations may be related to greater risk of expropriation, higher uncertainty on future returns from investments, lower maintenance expenditures in those countries, associated with greater corruption, exactly factors linked with institutions. For instance, Tanzi and Davoodi (1997) showed that higher corruption is associated with lower expenditures on operations and maintenance of physical capital, which calls for a relationship between institutions and the depreciation of physical capital, exactly the link that we uncover.

We define institutions as being associated with the concept of social infrastructure as in the work of Hall and Jones (1999, pp.84). For these authors social infrastructure is composed by "...institutions and government policies that determine the economic environment within which individuals accumulate skills, and firms accumulate capital and produce output". We use this definition of institutions in a broad sense, including both formal and informal institutions. While formal institutions include constitutional constraints, statutory rules, property rights, rule of law, and other political constraints; informal institutions arise from norms, culture, and customs, emerging spontaneously (Williamson, 2009). But formal institutions can contribute to economic growth only if they incorporate some of the principles established and agreed upon by informal institutions. This definition of informal institutions proposed by Williamson (2009) is closely related to the concept of social capital.[1] The notions of social infrastructure and trustworthy institutions are related to the notion of social capital. The notions of social capital and its most commonly used empirical proxy, trust, are related, and work as a substitute for the notion of property

[1] North (1990) and Knowles (2006) also emphasized the importance of informal institutions. Knowles (2006) relates the concepts of informal institutions and social capital, claiming that they are very similar. Berggren and Jordahl (2008) find an empirical positive relationship between the existence of a good legal structure and property rights (formal institutions in our definition) and the level of trust in economies (informal institutions in our definition).

rights (Aharonovitz *et al.*, 2009). There is a growing empirical literature relating institutions, social capital, and economic growth, namely Knack and Keefer (1997), Cuesta (2004), Beugelsdjik and van Schaik (2005), and Bjørnskov (2010), among others, pointing to a positive association between the mentioned variables, but still presenting diffuse results. In a model of endogenous growth, Strulik (2008) studies how social fractionalization and aggressiveness affect economic growth and show that civil conflict deters it.

In our work we focus on the positive role of institutions (social infrastructure) in preventing the depreciation of physical capital, which earlier empirical studies have uncovered, but works of theory have so far neglected. We build an endogenous model of economic growth with both physical and human capital accumulation in which we incorporate the important role of social infrastructure in facilitating physical capital preservation. Our main goal is to study an economic environment in which this feature is incorporated, focusing on the steady-state features and the transition path of the economy to the steady-state. The model will also allows us to assess the consequences of increasing this preservation effect both in transition and in equilibrium. The precise mechanisms according to which social infrastructure influences output (and hence economic growth) are underexplored in the literature.[2]

We fill this gap, proposing specific mechanisms according to which social infrastructure influences output by its direct effect on physical capital preservation. In the model, social infrastructure is modelled as a particular type of human capital allocation consisting of hours spent in several activities such as: petitions, influence groups, participation in informal networks that spread information, etc., i.e., activities of civic and community participation, which help to improve the level of civic rights, property rights, law and order, and ultimately the social infrastructure of a country. Through these effects social infrastructure reduces the incentive for rent seeking, corruption, predation, and cheating, and thus helps to preserve the existing physical capital stock of the economy. We analyze the economic consequences of such mechanisms. To this end and given the structure of the model, we use an innovative method of algebra in the economics field to study the existence and unicity of steady-states' solutions and equilibria - the Gröbner basis.

Section two presents some empirical evidence that motivates the paper. Section three presents the model. Then Section four characterizes the main results concerning steady-state equilibrium and its (local) stability. Section five presents simulation results for the transitional dynamics of the model when the effect of social infrastructure in investment is increased. In Section 6 we conclude.

2 Motivation

In this section we present empirical motivation for the relationship between social infrastructure and the accumulation of physical capital (investment). For that purpose we found two proxies that could be interpreted as social infrastructure - The Social Capital Index of the Prosperity Index from the Legatum Institute and the Social Capital Index from Hall and Jones (1999).

Figure 1 shows the relationship between the Social Capital Index 2010 from the Prosperity Index from the Legatum Institute and the Social Capital Index from Hall and Jones (1999) and Investment *per capita*, for about 120 countries.[3] Both panels in the figure show a positive relationship between Investment and

[2]Chin and Chou (2004) also model social infrastructure in a growth model, but in their model this variable affects the division of time between productive and non-productive activities. In our model it affects physical capital accumulation.

[3]The Social Capital Index 2010 was taken from the Legatum Institute website (http://www.prosperity.com/) and data for Investment (*per capita* and share of GDP in constant 2005 prices) were taken from the Penn World Tables, version 7.0.

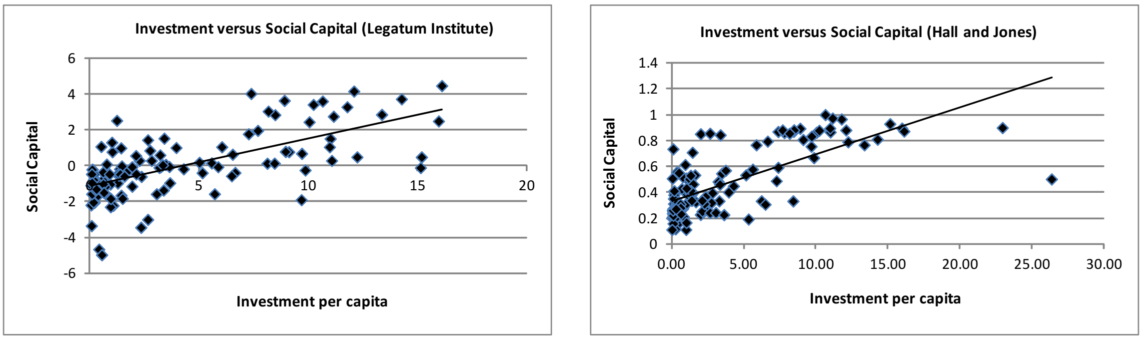

Figure 1: Relationship between Social Capital and Investment

Social Capital, empirically supporting the theorectical modelling followed in this paper, i.e., modelling social infrastructure as a positive effect in physical capital investment.

3 Model

We build an endogenous model of economic growth with both physical and human capital accumulation in which we incorporate the important role of social infrastructure in facilitating physical capital preservation. Human capital has different uses: it is employed in the production of the final good, in school attendance, which is the main input to the accumulation of new human capital, and it is also employed in the formation of social infrastructure. Physical capital is used in the production of the final good and social infrastructure facilitates the preservation of physical capital by decreasing its depreciation.

A crucial feature of the model is that there is no market for social infrastructure. Social infrastructure arises from the civic engagement of people and as a result provides utility. Also, households can help build and improve social infrastructure through allocating time to activities of civic and community participation. This follows the notion of *bonding* social capital in Beugelsdijk and Smulders (2009). However, firms also benefit from social infrastructure, since solid and trustful institutions can be a helpful production factor. This approach mimics both the notion of *bridging* social capital in the Beugelsdijk and Smulders (2009) article and the notion of *civic* capital in Guiso *et al.* (2010).

3.1 Production Factors and Final Goods

3.1.1 Capital Accumulation

Individual human capital can be divided into skills allocated to different activities (as in Lucas, 1988). Thus, skills can be allocated to the final good production (H_Y), to school attendance (H_H), and to the building and improving of social infrastructure (H_S). Assuming that the different human capital activities are not done cumulatively, we have:

$$K_H = H_Y + H_H + H_S. \tag{3.1}$$

This restriction can be written in shares of human capital utilization as $1 = u_Y + u_H + u_S$, with $u_Y = H_Y/K_H$, $u_H = H_H/K_H$ and $u_S = H_S/H_Y$.

As in the literature that began with Arnold (1998), in this model human capital is the "ultimate" source of growth. To have endogenous growth, one should have non-decreasing returns in the human capital production function, regardless of the inputs to human capital that are considered. Human capital

K_H is accumulated using human capital allocated to school attendance according to:

$$\dot{K}_H = \xi H_H \tag{3.2}$$

where $\xi > 0$ is a parameter that measures productivity in school attendance.

The accumulation of physical capital (K_P) arises through production that is not consumed, and is subject to depreciation:

$$\dot{K}_P = Y - C - \delta_P \left(1 - \sigma \frac{H_S}{K_H}\right) K_P \tag{3.3}$$

where Y denotes production of final goods, C is consumption, δ_P represents depreciation of physical capital, σ is the effect of social infrastructure in decreasing physical capital depreciation, and $\frac{H_S}{K_H} = u_S$ is the share of human capital in building and improving social infrastructure. Note that the constraint $\sigma u_S < 1$ must be satisfied to allow for a positive depreciation of physical capital.[4]

3.1.2 Final Good Production

The final good is a homogeneous one, produced with a Cobb-Douglas technology:

$$Y = K_P^{\beta} H_Y^{1-\beta}, 0 < \beta < 1 \tag{3.4}$$

where β is the share of physical capital in the final good production. If we substitute this equation into (3.3) physical capital is accumulated according to $\dot{K}_P = K_P^{\beta} H_Y^{1-\beta} - C - \delta_P(1 - \sigma u_S) K_P$. This means that the output-capital ratio can be written as $\frac{Y}{K_P} = \left(\frac{H_Y}{K_P}\right)^{1-\beta} = \left(\frac{K_H}{K_P}\right)^{1-\beta} u_Y^{1-\beta}$. Renaming $v_H = \frac{K_H}{K_P}$, we obtain:

$$\frac{Y}{K_P} = (v_H u_Y)^{1-\beta} \tag{3.5}$$

Similarly, we define $u_C = \frac{C}{K_P}$.

The markets for purchased production factors are assumed to be competitive. However, we assume that the firm cannot buy social infrastructure, as there is, in effect, no market for it. Social infrastructure is treated here as exogenous for the firm, although it affects the accumulation of physical capital.

From this problem we know that returns on production are as follows:

$$W_H = \frac{(1-\beta) Y}{H_Y} \tag{3.6}$$

$$r = \frac{\beta Y}{K_P} \tag{3.7}$$

where W_H is the market wage of workers and r is the rate of return of physical capital.

3.2 Consumers

We assume that households benefit directly from socializing, specifically engaging in civic activities. This follows the concept of bonding (as, for example, in Beugelsdijk and Smulders, 2009). Hence, household

[4] As we discuss above, we consider that social infrastructure is acting in order to preserve physical capital, decreasing its net depreciation rate. However, we would obtain similar results if we considered a direct and positive effect of social infrastructure on investment.

preferences specifies time spent in building and improving social infrastructure, along with consumption, as arguments of the intertemporal utility function:

$$U(C_t, H_{S_t}) = \frac{\tau}{\tau - 1} \int_0^{\infty} \left(C_t H_{St}^{\psi}\right)^{\frac{\tau-1}{\tau}} e^{-\rho t} dt \tag{3.8}$$

where ψ represents the preference for social infrastructure and ρ is the utility discount rate.[5]

In the market economy both consumers and firms make choices that maximize, respectively, their own utility or profits.[6] Consumers maximize their intertemporal utility function subject to the budget constraint:

$$\dot{a} = (r - \delta_p(1 - \sigma u_s))a + W_H (K_H - H_H - H_S) - C \tag{3.9}$$

where a represents the household's physical assets. The market price for the consumption good is normalized to 1. Since it is making an intertemporal choice, the household also takes into account equation (3.2), i.e., human capital accumulation.

The choice variables for the consumers are C, H_H, and H_S, so the first-order conditions for the consumer problem yield:

$$\frac{\partial U}{\partial C} = \lambda_a \tag{3.10}$$

$$\xi \lambda_H = \lambda_a W_H \tag{3.11}$$

$$\frac{\partial U}{\partial H_S} = \lambda_a W_H \tag{3.12}$$

as well as:

$$\frac{\dot{\lambda}_a}{\lambda_a} = \rho + \delta_P(1 - \sigma u_s) - r \tag{3.13}$$

$$\frac{\dot{\lambda}_H}{\lambda_H} = \rho - \xi \tag{3.14}$$

where λ_a is the co-state variable for the budget constraint and λ_H is the co-state variable for the stocks of human capital. Finally $\frac{\partial U}{\partial C} = C^{-1/\tau} H_S^{\psi \frac{\tau-1}{\tau}}$ and $\frac{\partial U}{\partial H_S} = \psi C^{\frac{\tau-1}{\tau}} H_S^{\psi \frac{\tau-1}{\tau} - 1}$.

The transversality conditions are: $\lim\limits_{t \to \infty} \lambda_a a e^{-\rho t} = 0$ and $\lim\limits_{t \to \infty} \lambda_H K_H e^{-\rho t} = 0$.

3.3 The Economy Dynamics

Using (3.10), (3.13), (3.5), and (3.3), we obtain g_{u_C}:

$$\begin{aligned} g_{u_C} = {} & (\tau - 1)\psi g_{u_S} + (\tau - 1)\psi \xi (1 - u_Y) - (1 - \tau\beta)(u_Y v_H)^{1-\beta} + \\ & + (1 - \tau)\delta_P + u_C - ((\tau - 1)\psi\xi + (1 - \tau)\sigma\delta_P) u_S - \tau\rho. \end{aligned} \tag{3.15}$$

Resorting to (3.2), (3.1), and (3.3), the expression for g_{v_H} becomes:

$$g_{v_H} = \xi(1 - u_Y) - (u_Y v_H)^{1-\beta} + u_C + \delta_P - (\xi + \sigma\delta_P) u_S \tag{3.16}$$

[5] The t subscripts are dropped hereinafter for ease of notation.

[6] In this section we are working with variables for individual consumers.

From (3.11) and (3.6), we obtain the growth rate of u_Y:

$$g_{u_Y} = 1/\beta\frac{\dot{\lambda}_a}{\lambda_a} + g_{K_P} - 1/\beta\frac{\dot{\lambda}'_H}{\lambda'_H} - \xi(1-u_Y-u_S) \tag{3.17}$$

and from (3.13) and (3.14) we reach:

$$g_{u_Y} = \frac{\delta_P}{\beta}(1-\sigma u_S) - (u_Y v_H)^{1-\beta} + \frac{\xi}{\beta} + g_{K_P} - \xi(1-u_Y-u_S). \tag{3.18}$$

Replacing g_{K_P} by its expression (3.3), we then obtain:

$$g_{u_Y} = \left(\frac{1}{\beta}-1\right)\delta_P(1-\sigma u_S) - \xi(1-u_Y-u_S) - u_C + \frac{\xi}{\beta}. \tag{3.19}$$

Finally, from (3.11) and (3.12), we compute $\psi C^{\frac{\tau-1}{\tau}} H_S^{\psi\frac{\tau-1}{\tau}-1} = \lambda_a W_H$. Using (3.10) and (3.6) we obtain $u_S = \frac{\psi}{1-\beta}\frac{C}{Y}u_Y$, which is easily converted into the static equation:

$$u_S = \frac{\psi}{1-\beta}\frac{u_C}{(u_Y v_H)^{1-\beta}}u_Y \tag{3.20}$$

We now have a system of three differential equations on u_C, u_Y, and v_H with a static equation on u_S, which, using $\bar{z} = v_H^{1-\beta}u_Y^{1-\beta}$, can be written as:

$$\begin{aligned}
g_{u_C} &= (\tau-1)\psi g_{u_S} + (\tau-1)\psi\xi(1-u_Y) - (1-\tau\beta)\bar{z} + \\
&\quad +(1-\tau)\delta_P + u_C - ((\tau-1)\psi\xi + (1-\tau)\sigma\delta_P)u_S - \tau\rho \\
g_{v_H} &= \xi(1-u_Y) - \bar{z} + u_C + \delta_P - (\xi+\sigma\delta_P)u_S \\
g_{u_Y} &= \left(\frac{1}{\beta}-1\right)\delta_P(1-\sigma u_S) - -\xi(1-u_Y-u_S) - u_C + \frac{\xi}{\beta} \\
u_S &= \frac{\psi u_C}{(1-\beta)\bar{z}/u_Y}
\end{aligned} \tag{3.21}$$

4 Steady-State

The solution of economic growth models is often characterized as a set of multivariate polynomial equations, resulting from setting growth rates of stationary variables to zero. The system characterizing the decentralized equilibrium is a parametrized system of *four* variables, *four* equations, and *seven* parameters. If one seeks to solve the system with the usual techniques (e.g. Gauss' elimination), one obtains an equation in only one variable that is too complex to handle and analyze.

In the last 30 years, computational algebraic geometry has seen considerable advances in methods that solve polynomial systems. The method of Gröbner basis is a powerful example of this progress. In fact, one can find in the literature very recent applications of Gröbner basis in modern economic models. For example, Kubler and Schmedders use them to study the multiplicity of equilibria (see Kubler and Schmedders, 2010a), and to compute the equilibrium correspondence for exchange economies with semi-algebraic preferences (see Kubler and Schmedders, 2010b).[7]

[7]For the basic definitions and concepts on algebraic geometry and Gröbner bases these two papers provide an introduction to this subject. We refer the reader to the textbook Cox *et al.* (1997) for more profound reading on this topic.

Let $K[x_1,\ldots,x_n]$ be the ring of polynomials in n variables x_i with coefficients in a field K. The main idea behind the Gröbner basis technique is the following: given a set S of polynomials in $K[x_1,\ldots,x_n]$ that describes the problem in hand, one transforms S into another set T of polynomials of much simpler form, called a Gröbner basis, such that S and T are "equivalent", i.e., they have the same set of solutions. Thus, difficult problems for general S become "easier" for Gröbner basis T. For linear polynomials, the Gröbner basis algorithm specializes to Gauss' algorithm, whereas for univariate polynomials it specializes to Euclid's algorithm.

One of the main advantages of this algorithm is that we can compute Gröbner basis for parametrized polynomials. In particular, one can compute the number of equilibria for entire classes of economic models (or bounds for this number), search for specific parameter values for which there are multiple equilibria, or prove that equilibria are unique for all parameter values in a given set. However, one must be aware that there may exist some parameters for which the corresponding Gröbner basis obtained is not the correct one. More precisely, Gröbner basis behave nicely for most (but not all) values of the parameters in the following sense: there is a proper subvariety $W \subset \mathbb{R}^m$ (where m is the number of parameters of the system) such that the Gröbner basis obtained is the same when the parameters take values in $\mathbb{R}^m - W$ (see Cox *et al.*, 1997, Chapter 6, §3).

There exist special software systems that are based mainly on the Gröbner basis technique. In this paper we use the computer algebra system Singular (Decker *et al.*, 2011). In Kubler and Schmedders (2010a) the reader can find some simple examples of how to compute Gröbner basis with Singular.

Finally, we would like to remark that, in general, for a given set of polynomials there exist infinitely many Gröbner basis. However, when we fix a monomial order, there is a unique "reduced" Gröbner basis (a minimal basis better than the others). Under some mild assumptions, this basis is of a very simple and special form (see Kubler and Schmedders, 2010a and 2010b). In the present work it has proved sufficient for our goals to compute the reduced Gröbner basis, without checking the mild hypotheses referred to.

As explained above, the Gröbner basis' method allows us to simplify the system. Even so, the analysis of this simpler system still involves *seven* parameters. The computation of the number of equilibria can lead to very tedious and long calculations and, most probably, inconclusive results.

In order to obtain a sensible analysis of the steady-state, we must calibrate our model with sensible values for the parameters, usually used in endogenous growth theory and keep free the most important parameters linked with social infrastructure, the focus of our paper. Some parameters in our model are quite standard in the literature: the intertemporal substitution parameter ($\tau = 0.5$), the intertemporal discount factor ($\rho = 0.02$), and the share of physical capital in income ($\beta = 0.36$), so we shall not discuss them. For other parameters there are a range of plausible values, although most of them present typical values that are most used in the literature: the depreciation rate (δ_P), which we set to be 0.05 and the productivity of school attendance (ξ), which we set to be 0.05. We begin by studying steady-state solutions in which we calibrate all the parameters except those directly related with social infrastructure, ψ and σ, which we keep free. We then move from this general approach to more specific solutions in which we calibrate ψ and allow the parameter that governs the impact of social infrastructure on investment $-\ \sigma -$, the main mechanism analyzed in this paper, to be free. We assume that the weight the consumer attributes to social infrastructure is lower that the weight attributed to consumption, thus we implement solutions with ψ equal to 0.1, 0.5, and 0.9.

4.1 Steady-State for Free Social Infrastructure Parameters (ψ and σ)

The system of equations describing the decentralized equilibrium (3.21) when all parameters but ψ and σ are calibrated is:

$$\begin{cases} u_C + (0.025\psi - 0.025\sigma)u_S + (0.025\psi)u_Y - 0.82\bar{z} - 0.025\psi + 0.015 = 0 \\ u_C - (0.05 + 0.05\sigma)u_S - 0.05u_Y - \bar{z} + 0.1 = 0 \\ u_C + (4/45\sigma - 0.05)u_S - 0.05u_Y - 8/45 = 0 \\ \psi u_C u_Y - 0.64u_S\bar{z} = 0 \end{cases} \tag{4.1}$$

Singular gives us the following Gröbner basis for the above system of polynomial equations:

$$\begin{aligned} g_1(\bar{z}) &= \bar{z}^2 + \tfrac{80\sigma\psi^2+112\sigma\psi-115\psi^2-417\psi-320}{576\psi^2+1728\psi+1152}\bar{z} + \tfrac{25\sigma\psi^3+70\sigma\psi^2+49\sigma\psi-50\psi^3-170\psi^2-140\psi}{2304\psi^3+11520\psi^2+18432\psi+9216}; \\ g_2(u_Y,\bar{z}) &= u_Y - \tfrac{36}{5\sigma}\bar{z} + \tfrac{-5\sigma\psi-7\sigma+10\psi+20}{5\sigma\psi+10\sigma}; \\ g_3(u_S,\bar{z}) &= u_S + \tfrac{36}{5\sigma}\bar{z} - \tfrac{2}{\sigma}; \\ g_4(u_C,\bar{z}) &= u_C - \tfrac{16}{25}\bar{z} - \tfrac{5\psi+7}{100\psi+200}. \end{aligned} \tag{4.2}$$

This means that the system (4.1) is equivalent to the simplified system:

$$g_1(\bar{z}) = g_2(u_Y,\bar{z}) = g_3(u_S,\bar{z}) = g_4(u_C,\bar{z}) = 0.$$

Clearing denominators in the first and second equations, we can write $Ag_1(\bar{z}) = A\bar{z}^2 + B\bar{z} + C$ and $Dg_2(u_Y,\bar{z}) = Du_Y + E\bar{z} + F$, where the coefficients A, B, C, D, E, and F are functions whose variables are the parameters ψ and σ, the system has the following recursive form solution:

$$\begin{cases} \bar{z} = \frac{-B\pm\sqrt{B^2-4AC}}{2A} \\ u_Y = -\frac{E\bar{z}+F}{D} \\ u_S = -\frac{36}{5\sigma}\bar{z} + \frac{2}{\sigma} \\ u_C = \frac{16}{25}\bar{z} + \frac{5\psi+7}{100\psi+200} \end{cases}$$

We can rewrite the system in the following way (after substituting the value of $\bar{z}$ in all equations):

$$\begin{cases} \bar{z} = \frac{-B\pm\sqrt{B^2-4AC}}{2A} \\ u_Y = \frac{-2AF+BE\mp E\sqrt{B^2-4AC}}{2AD} \\ u_S = \frac{10A+18B\mp 18\sqrt{B^2-4AC}}{5\sigma A} \\ u_C = \frac{5\psi A-32\psi B+7A-64B\pm 32(\psi+2)\sqrt{B^2-4AC}}{100\psi A+200A} \end{cases} \tag{4.3}$$

Our goal is to determine, for each ψ and σ, the number of real positive solutions for this system. Although we have a general solution as above, each variable is expressed by a complicated function depending on ψ and σ. For instance, $B^2 - 4AC$ is a polynomial of degree 8 in these parameters. However, since all variables depend only on *two* parameters, we can study their expressions to obtain numerical approximations of when they are real and positive.

We first analyze when $\bar{z}$ is real and positive. We can determine when $B^2 - 4AC > 0$ and $\sigma \in]0, 10[$.

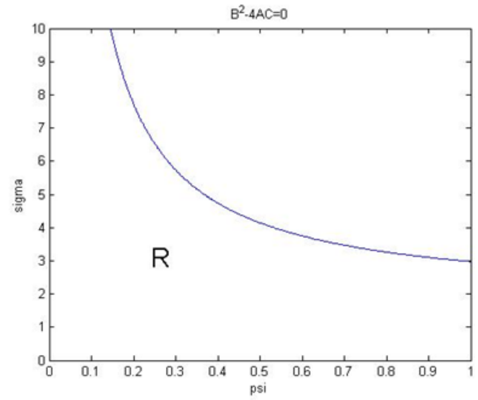

Figure 2: Region R for which $\bar{z}$ is real and $\bar{z}_1$ is real and positive

The line in Figure 2, $B^2 - 4AC = 0$, divides the plane into two regions. The one labeled by R represents the set of (almost) all values of ψ and σ for which $B^2 - 4AC > 0$, and thus we have real values for $\bar{z}$, when $0 < \psi < 1$ and $0 < \sigma < 10$. These intervals for the social infrastructure parameters are based on quite weak assumptions. The first one ($0 < \psi < 1$) means that social infrastructure contributes (positively) to utility but weights less than consumption (which weights 1); thus ψ measures the relative welfare-substitutability between social infrastructure and consumption. The second interval ($0 < \sigma < 10$) means that social infrastructure preserves physical capital (the main assumption of this article) – as $0 > \sigma$ would clearly be dismissed by data – and $\sigma < 10$ prevents the overall effect of social infrastructure share in the growth rate of capital from exceeding one, i.e., $1 > \partial g_K / \partial u_S > 0$. This restriction also keeps the value of the overall effect of social infrastructure in preserving physical capital within a reasonable interval, even though for higher values of that interval it would be possible that the strength of the social infrastructure effect offsets the negative effect of depreciation.[8] We will thus focus only in this "admissible" region, where $\bar{z}$ takes real values.

One can easily check that $A > 0$ for all ψ and σ, and $B < 0$ in the region R. So, we conclude that $\bar{z}_1 = \frac{-B+\sqrt{B^2-4AC}}{2A}$ is always positive for general values of ψ and σ in R.

Note that given any $\bar{z} > 0$ (i.e. given general values of $\psi, \sigma \in R$), the system has at least one real solution $(\bar{z}, u_Y, u_S, u_C)$. We wish to examine the situation when this solution is positive and determine how many positive solutions the system has.

Consider $\bar{z}_1 \in R$. From (4.3), it is easy to see that $u_Y > 0$ if and only if the numerator $-2AF + BE - E\sqrt{B^2 - 4AC} > 0$. The study of this function allows us to conclude that it is positive for general values of ψ and σ in R. This guarantees us that when $z = z_1$, there is always a positive real solution u_Y.

The following step is to evaluate the sign of the variable u_S when $\bar{z} = \bar{z}_1$. As before, u_S is positive if and only if its numerator is positive. This holds in R and hence, there is a positive solution $u_S > 0$ in R.

Finally, from the expression obtained for u_C, we see that $u_C > 0$ if and only if $5\psi A - 32\psi B + 7A - 64B \pm 32(\psi + 2)\sqrt{B^2 - 4AC} > 0$. Studying this two-variable function, we see that $u_C > 0$ for general values of σ and ψ in the region below R when $z = z_1$ (Figure 3 shows the region where $u_C > 0$, clearly containing region R shown in Figure 2).

[8]Below, we will note that u_S would be around 0.3 even for values of σ approaching 10. This means that $(1 - \sigma u_S)$ would reach -2 with $\sigma = 10$, corresponding to adding 10% in the g_K, a quite high and unreasonable value. With $\sigma < 10$ we limit the analysis to effects that are always lower than that.

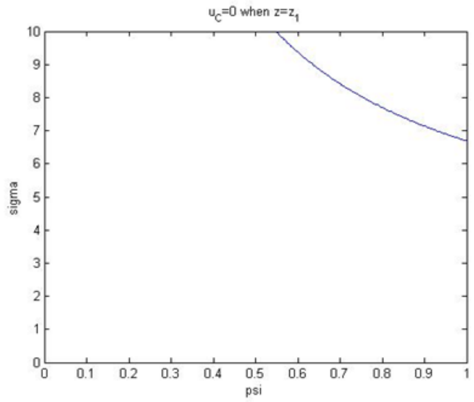

Figure 3: Region for which u_C is real and positive, when $\bar{z} = \bar{z}_1$

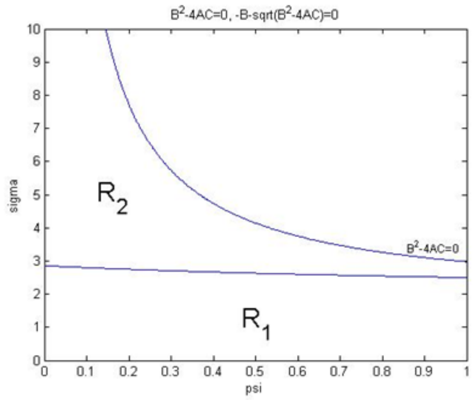

Figure 4: Region R_2 for which $\bar{z}_2$ is real and positive

Now, let us study the case when $\bar{z}_2 = \frac{-B-\sqrt{B^2-4AC}}{2A}$. Figure 4 shows us how the sign of $-B-\sqrt{B^2-4AC}$ changes inside R.

We see that $-B-\sqrt{B^2-4AC} = 0$ divides R into two smaller open regions. More precisely,

$$R = R_1 \cup R_2 \cup \{(\sigma, \psi) \in]0, 10[\times]0, 1[: -B-\sqrt{B^2-4AC} = 0\}.$$

In the region R_1 one has $-B-\sqrt{B^2-4AC} < 0$, whereas in R_2 one has $-B-\sqrt{B^2-4AC} > 0$. Therefore, $\bar{z}_2 > 0$ if and only if ψ and σ belong to the region R_2. In this case, when we study the sign of the corresponding u_Y (i.e., when $\bar{z} = \bar{z}_2$) we have $u_Y > 0$ in R_2 in Figure 4.[9] Therefore, there is a positive solution for u_Y when $\sigma, \psi \in R_2$ (The line dividing R_1 and R_2 describes the set of points where $u_Y = 0$ in R).

Studying the functions defining u_S and $u_C > 0$ when $\bar{z} = \bar{z}_2$, we conclude that both are positive for general values of $\sigma, \psi \in R_2$ (in fact, they are positive in R).

[9]It is sufficient to look at the numerator since the denominator is always positive. The same holds when we study u_S and u_C.

We can now conclude our study. For almost all $\psi, \sigma \in R_1$, the system (4.1) has a unique positive solution:

$$\begin{cases} \overline{z} = \frac{-B+\sqrt{B^2-4AC}}{2A} \\ u_Y = \frac{-2AF+BE-E\sqrt{B^2-4AC}}{2AD} \\ u_S = \frac{10A+18B-18\sqrt{B^2-4AC}}{5\sigma A} \\ u_C = \frac{5\psi A-32\psi B+7A-64B+32(\psi+2)\sqrt{B^2-4AC}}{100\psi A+200A} \end{cases}$$

For generic values of ψ and σ in the region R_2, the system has two positive solutions:

$$\begin{cases} \overline{z} = \frac{-B+\sqrt{B^2-4AC}}{2A} \\ u_Y = \frac{-2AF+BE-E\sqrt{B^2-4AC}}{2AD} \\ u_S = \frac{10A+18B-18\sqrt{B^2-4AC}}{5\sigma A} \\ u_C = \frac{5\psi A-32\psi B+7A-64B+32(\psi+2)\sqrt{B^2-4AC}}{100\psi A+200A} \end{cases} \vee \begin{cases} \overline{z} = \frac{-B-\sqrt{B^2-4AC}}{2A} \\ u_Y = \frac{-2AF+BE+E\sqrt{B^2-4AC}}{2AD} \\ u_S = \frac{10A+18B+18\sqrt{B^2-4AC}}{5\sigma A} \\ u_C = \frac{5\psi A-32\psi B+7A-64B-32(\psi+2)\sqrt{B^2-4AC}}{100\psi A+200A} \end{cases}$$

The most interesting result in this subsection is that we can define the regions in the space (ψ, σ) in which the equilibrium is unique and the regions in which there are two different feasible equilibria. Unicity is obtained for low values of σ ($\lesssim 2.8$) and for almost all values of ψ, as we can see in Figure 4, where R_1 is the region in which there is only a single positive steady-state and R_2 is the region in which there are two positive steady-states.

4.1.1 Finding W

As mentioned in the introduction to this section, there is a proper subvariety $W \subset \mathbb{R}^2$ such that when parameters ψ and σ take values outside W Gröbner basis behave nicely, i.e., the polynomials obtained from $g_1, \dots, g_4$ by choosing values for ψ and σ are still a Gröbner basis for the ideal generated by the polynomials obtained from the original polynomials in equations (4.1). We will determine W in order to ensure that the Gröbner basis defined above is the correct one for this problem. This calculation is not straightforward, as the literature on the subject mentiuoned above points out.

In Cox *et al.*, 1997, Chapter 6, §3, exercises 7–9, we have a set of guidelines to compute W, which we will follow here.

Let $f_1, \dots, f_4$ be the polynomials

$$\begin{aligned} f_1 &= u_C + (0.025\psi - 0.025\sigma)u_S + (0.025\psi)u_Y - 0.82\overline{z} - 0.025\psi + 0.015; \\ f_2 &= u_C - (0.05 + 0.05\sigma)u_S - 0.05u_Y - \overline{z} + 0.1; \\ f_3 &= u_C + (4/45\sigma - 0.05)u_S - 0.05u_Y - 8/45; \\ f_4 &= \psi u_C u_Y - 0.64u_S\overline{z}. \end{aligned}$$

Let I be the ideal of $\mathbb{C}(\psi, \sigma)[u_C, u_S, u_Y, \overline{z}]$ generated by the polynomials f_1, f_2, f_3, and f_4. Consider the lexicographical ordering for monomials with

$$u_C > u_S > u_Y > \overline{z}.$$

A reduced Gröbner basis for I is

$$
\begin{aligned}
h_1 &= \bar{z}^2 + \tfrac{80\sigma\psi^2+112\sigma\psi-115\psi^2-417\psi-320}{576\psi^2+1728\psi+1152}\bar{z} + \tfrac{25\sigma\psi^3+70\sigma\psi^2+49\sigma\psi-50\psi^3-170\psi^2-140\psi}{2304\psi^3+11520\psi^2+18432\psi+9216};\\
h_2 &= u_Y - \tfrac{36}{5\sigma}\bar{z} + \tfrac{-5\sigma\psi-7\sigma+10\psi+20}{5\sigma\psi+10\sigma};\\
h_3 &= u_S + \tfrac{36}{5\sigma}\bar{z} - \tfrac{2}{\sigma};\\
h_4 &= u_C - \tfrac{16}{25}\bar{z} - \tfrac{5\psi+7}{100\psi+200}.
\end{aligned}
$$

We can now see that f_1, f_2, and f_3 are monic polynomials for the monomial ordering we considered. If we divide f_4 by ψ, we obtain a monic polynomial, as well. Being a reduced Gröbner basis, polynomials h_1, h_2, h_3, and h_4 are also monic. Let us consider all denominators present in the coefficients of polynomials f_1, f_2, f_3, $\frac{1}{\psi}f_4$, h_1, h_2, h_3, and h_4 (coefficients are elements of $\mathbb{C}(\psi,\sigma)$). They are:

$$
\begin{aligned}
d_1 &= \psi; & d_4 &= 5\sigma;\\
d_2 &= 576\psi^2 + 1728\psi + 1152; & d_5 &= 5\sigma\psi + 10\sigma;\\
d_3 &= 2304\psi 3 + 11520\psi 2 + 18432\psi + 9216; & d_6 &= 100\psi + 200.
\end{aligned}
$$

When we consider these polynomials in the ring $\mathbb{C}[\psi,\sigma]$, their least common multiple can be computed using library `poly.lib` (Bachmann *et al.*, 2011) in Singular. It is

$$d = \sigma\psi(\psi^3 + 5\psi^2 + 8\psi + 4).$$

Now let $\tilde{I}$ be the ideal of $\mathbb{C}[u_C, u_S, u_Y, \bar{z}, \phi, \sigma]$ generated by the polynomials f_1, f_2, f_3, and f_4. Let h_1', h_2', h_3', and h_4' be the polynomials we obtain by clearing denominators in h_1, h_2, h_3, and h_4, respectively. These polynomials are:

$$
\begin{aligned}
h_1' &= (2304\psi^3 + 11520\psi^2 + 18432 + 9216)\bar{z}^2\\
&\qquad + (320\sigma\psi^3 + 1088\sigma\psi^2 + 896\sigma\psi - 460\psi^3 - 2588\psi^2 - 4616\psi - 2560)\bar{z}\\
&\qquad + (25\sigma\psi^3 + 70\sigma\psi^2 + 49\sigma\psi - 50\psi^3 - 170\psi^2 - 140\psi);\\
h_2' &= (5\sigma\psi + 10\sigma)u_Y - (36\psi + 72)\bar{z} - 5\sigma\psi - 7\sigma + 10\psi + 20;\\
h_3' &= 5\sigma u_S + 36\bar{z} - 10;\\
h_4' &= (100\psi + 200)u_C - (64\psi + 128)\bar{z} - 5\psi - 7.
\end{aligned}
$$

By computing a Gröbner basis for $\tilde{I}$, we can easily see that all polynomials h_1', h_2', h_3', and h_4' are in $\tilde{I}$, and we can therefore conclude that if W is the variety defined by d in $\mathbb{R}^2$, then for all $(\psi,\sigma) \in \mathbb{R}^2 \setminus W$ the Gröbner basis specialize well.

Note that d vanishes for $\sigma = 0$, $\psi = 0$ or negative values of ψ. All these values are excluded in the present context, so for the values relevant herein, the Gröbner basis computed above will specialize well.

4.2 Steady-State for the Free Effect of Social Infrastructure on Investment (σ)

The main focus of this paper is to study an endogenous growth model in which we incorporate an effect of social infrastructure in preserving physical capital. Thus, we wish to detail the steady-state solutions for some given values of the effect of social infrastructure in utility (ψ) and only for a free effect of social infrastructure in investment (σ). We use three values for $\psi : 0.5, 0.1$, and 0.9.

Replacing $\psi = 0.5$ in system (4.1) and computing its reduced Gröbner basis is the same as replacing it in the reduced Gröbner basis above, as we saw in the last section. It yields the following:

$$\begin{aligned} g_1(\bar{z}) &= \bar{z}^2 + \left(\tfrac{19}{540}\sigma - \tfrac{743}{2880}\right)\bar{z} + \tfrac{361}{172800}\sigma - \tfrac{19}{3456} \\ g_2(u_Y,\bar{z}) &= u_Y - \tfrac{36}{5\sigma}\bar{z} + \tfrac{50-19\sigma}{25\sigma} \\ g_3(u_S,\bar{z}) &= u_S + \tfrac{36}{5\sigma}\bar{z} - \tfrac{2}{\sigma} \\ g_4(u_C,\bar{z}) &= u_C - \tfrac{16}{25}\bar{z} - \tfrac{19}{500} \end{aligned} \tag{4.4}$$

The solution of this system is:

$$\begin{cases} \bar{z} = \frac{-304\sigma+2229\pm\sqrt{92416\sigma^2-1979040\sigma+6610041}}{17280} \\ u_Y = \frac{36}{5\sigma}\bar{z} - \frac{50-19\sigma}{25\sigma} \\ u_S = -\frac{36}{5\sigma}\bar{z} + \frac{2}{\sigma} \\ u_C = \frac{16}{25}\bar{z} + \frac{19}{500} \end{cases}$$

or, equivalently:

$$\begin{cases} \bar{z} = \frac{-304\sigma+2229\pm\sqrt{92416\sigma^2-1979040\sigma+6610041}}{17280} \\ u_Y = \frac{1520\sigma-2571\pm\sqrt{92416\sigma^2-1979040\sigma+6610041}}{2400\sigma} \\ u_S = \frac{304\sigma+2571\mp\sqrt{92416\sigma^2-1979040\sigma+6610041}}{2400\sigma} \\ u_C = \frac{-304\sigma+3255\pm\sqrt{92416\sigma^2-1979040\sigma+6610041}}{27000} \end{cases}$$

Comparing equilibria in the case in which they both exist, we can see that one is characterized with a higher allocation of human capital to the final good production and high consumption to capital ratio while the economy invests less in social infrastructure, while the other is characterized by lower allocation to the final good production and consumption and better institutional environment. There is thus a trade-off between present and future, determined by allocation of human resources to build social infrastructure. In Figure 5, we see that for $\sigma \in R_1 =]0;a[$, where $a \approx 2.6316$, there is exactly one positive solution $\bar{z}$, namely:

$$\bar{z} = \frac{-304\sigma+2229+\sqrt{92416\sigma^2-1979040\sigma+6610041}}{17280},$$

whereas when $\sigma \in R_2 =]a;b[$, where $b \approx 4.1406$, there are two possible positive solutions:

$$\bar{z} = \frac{-304\sigma+2229\pm\sqrt{92416\sigma^2-1979040\sigma+6610041}}{17280}.$$

When $\sigma \in]b;10[$, $\bar{z}$ is a complex solution. Furthermore, we see which values $\bar{z}$ takes when σ varies between 0 and 10. The graphs in Figure 5 show the values for $\bar{z}$, u_Y, u_S, and u_C.

Note that all results obtained are coherent with those obtained in the previous section. Suppose that $\sigma \in R_1 =]0;a[$. In this case, the only $\bar{z} > 0$ determines a unique admissible solution of the system, $(\bar{z}, u_Y, u_S, u_C)$, although the graphs in Figure 5 show us that there are two possible positive solutions for u_S

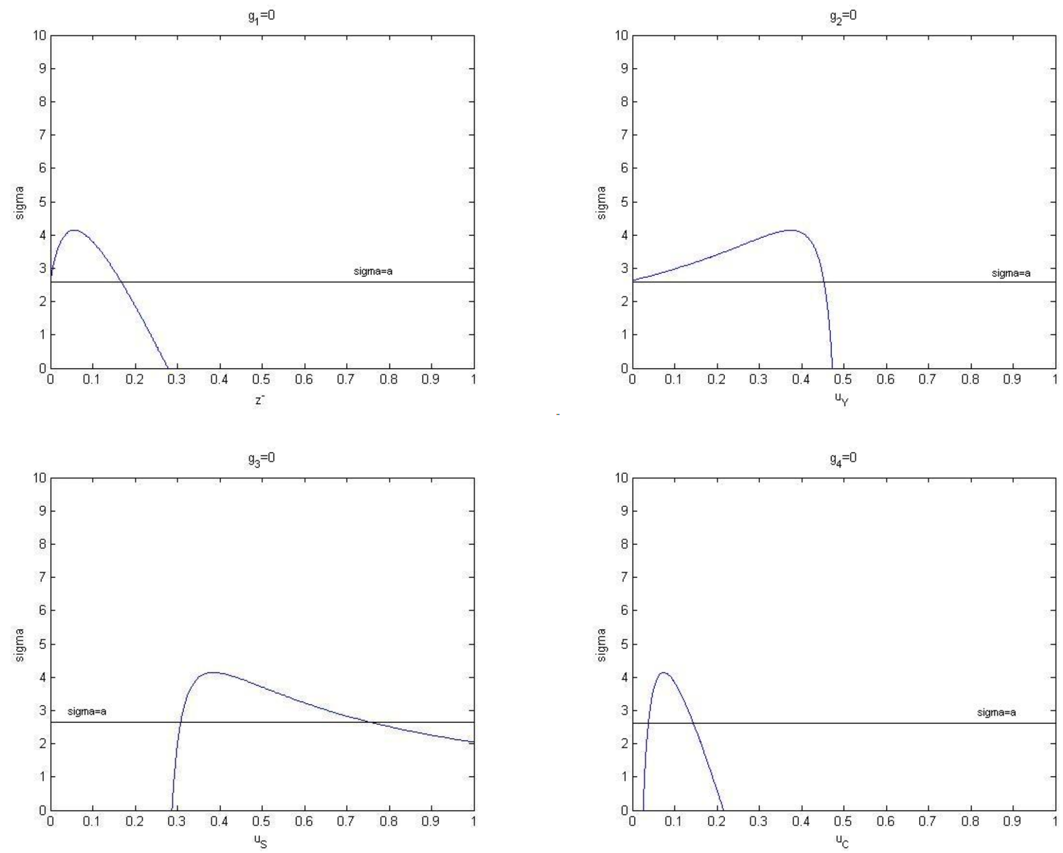

Figure 5: Real and Positive solutions of $\bar{z}$, u_Y, u_S and u_C for $\sigma \in]0;10[$ and $\psi = 0.5$.

and u_C (recall from the previous section that when $\bar{z} = \bar{z}_1$ all variables are positive for all $(\sigma, \psi) \in R_1$; but when $\bar{z} = \bar{z}_2$ only u_S and u_C are positive for all $(\sigma, \psi) \in R_1$).

On the other hand, if $\sigma \in R_2 =]a;b[$, we are able to check in figure 5 that a horizontal line above the line $\sigma = a$ and below $\sigma = b$ intersects $\bar{z}$, u_Y, u_S, and u_C at two points. This means that there are two solutions for the system (4.4).

Looking at Figure 5 gives us an idea about how reasonable this exercise is. In fact, we obtain an allocation of human capital to the final good that can be at most 0.5, an allocation of human capital to social infrastructure that can be around 0.3 (in the unique equilibrium or in one of the equilibria when there are two) and a consumption to capital ratio can be just above zero or nearly 0.2, also reasonable values. When analyzing the implications of the two equilibria solution, we easily reach the conclusion that the country with higher u_S, lower u_Y, u_C, and $\bar{z}$ would also have a lower Y/K_P. Whether the country with higher social infrastructure would have a higher income level than the one with lower infrastructure would depend on the level of K_P. However, this level would depend on, among other things, the efforts countries had made in order to improve σ, since an increase in σ will increase the growth rate of capital above the steady-state level and ultimately determine the income level of the country in each period. This draws attention to the study of transitional dynamics effects, which we present below.

The cases when $\psi = 0.1$ and $\psi = 0.9$ are studied in similar ways and give results that are analogous to the case when $\psi = 0.5$. Reduced Gröbner basis are, respectively:

$$\begin{aligned} g_1(\bar{z}) &= \bar{z}^2 + \left(\tfrac{25}{2772}\sigma - \tfrac{12095}{44352}\right)\bar{z} + \tfrac{625}{1241856}\sigma - \tfrac{125}{88704} \\ g_2(u_Y, \bar{z}) &= u_Y - \tfrac{36}{5\sigma}\bar{z} - \tfrac{5\sigma - 14}{7\sigma} \\ g_3(u_S, \bar{z}) &= u_S + \tfrac{36}{5\sigma}\bar{z} - \tfrac{2}{\sigma} \\ g_4(u_C, \bar{z}) &= u_C - \tfrac{16}{25}\bar{z} - \tfrac{1}{28} \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} g_1(\bar{z}) &= \bar{z}^2 + \left(\tfrac{115}{2204}\sigma - \tfrac{78845}{317376}\right)\bar{z} + \tfrac{13225}{4090624}\sigma - \tfrac{575}{70528} \\ g_2(u_Y, \bar{z}) &= u_Y + \tfrac{36}{5\sigma}\bar{z} + \tfrac{-23\sigma + 58}{29\sigma} \\ g_3(u_S, \bar{z}) &= u_S + \tfrac{36}{5\sigma}\bar{z} - \tfrac{2}{\sigma} \\ g_4(u_C, \bar{z}) &= u_C - \tfrac{16}{25}\bar{z} - \tfrac{23}{580} \end{aligned} \tag{4.6}$$

When $\psi = 0.1$, the solution of the system is:

$$\begin{cases} \bar{z} = \frac{-2800\sigma + 84665 \pm 5\sqrt{313600\sigma^2 - 26726560\sigma + 308458969}}{620928} \\ u_Y = \frac{11760\sigma - 17563 \pm \sqrt{313600\sigma^2 - 26726560\sigma + 308458969}}{17248\sigma} \\ u_S = \frac{560\sigma + 17563 \mp \sqrt{313600\sigma^2 - 26726560\sigma + 308458969}}{17248\sigma} \\ u_C = \frac{-560\sigma + 23863 \pm \sqrt{313600\sigma^2 - 26726560\sigma + 308458969}}{194040} \end{cases}$$

For $\sigma \in]0;10[$, $\bar{z}$ is always a real number. In this case, we have $R_1 =]0;c[$, with $c \approx 2.8$, and the system has only one positive solution, and $R_2 =]c;10[$, where we find two positive solutions of the system.

When $\psi = 0.9$, the solution of the system is

$$\begin{cases} \bar{z} = \frac{-480240\sigma+2286505\pm 5\sqrt{9225218304\sigma^2-131665479840\sigma+319626276025}}{18407808} \\ u_Y = \frac{309488\sigma-565355\pm\sqrt{9225218304\sigma^2-131665479840\sigma+319626276025}}{511328\sigma} \\ u_S = \frac{96048\sigma+565355\mp\sqrt{9225218304\sigma^2-131665479840\sigma+319626276025}}{511328\sigma} \\ u_C = \frac{-96048\sigma+685415\pm\sqrt{9225218304\sigma^2-131665479840\sigma+319626276025}}{5752440} \end{cases}$$

Now, $R_1 =]0;d[$, where $d \approx 2.5217$, and $R_2 =]d;e[$, where $e \approx 3.1015$. For $\sigma \in R_1$ the system has only one positive solution, while for $\sigma \in R_2$ there are two positive solutions. For $\sigma \in]e;10[$, $\bar{z}$ is not a real number.

This section divides the space of the effect of social infrastructure on investment according to the existence of steady-state and its unicity. There is unicity of the steady-state when the effect of social infrastructure on investment is relatively low ($0 < \sigma < 3$) and there are two feasible equilibria for values greater than 3 for this parameter. The precise value of σ below which there is a unique equilibrium does not change much when the weight of social infrastructure in utility changes from 0.1 to 0.9.

4.3 Stability

In this section we wish to study the stability around the steady-states presented above. This is important in order to know if the system converges to the steady-state, once deviating from it temporarily. To this end we linearize the system (3.21) around the steady-state (v_H^*, u_Y^*, u_C^*) and obtain the following:

$$\begin{pmatrix} \dot{v}_H \\ \dot{u}_Y \\ \dot{u}_C \end{pmatrix} = \begin{pmatrix} u_C + \frac{u_C u_Y (1-\tau)\psi(\xi\psi - \delta_P\sigma)}{\bar{z}(1-\beta)} & J_{12} & J_{13} \\ v_H - \frac{u_Y^{\beta} v_H^{\beta}(\xi+\delta_P\sigma)\psi}{(1-\beta)} & J_{22} & J_{23} \\ -u_Y\left(1 - \frac{u_Y\psi\Upsilon}{\bar{z}(1-\beta)\beta}\right) & -\frac{u_C u_Y^{1+\beta} v_H^{-2+\beta}\Upsilon\psi}{\beta} & \xi u_Y + \frac{u_C u_Y \Upsilon\psi}{\bar{z}(1-\beta)} \end{pmatrix} \begin{pmatrix} v_H - v_H^* \\ u_Y - u_Y^* \\ u_C - u_C^* \end{pmatrix}, \tag{4.7}$$

$$\begin{aligned} J_{12} &= -u_C\bar{z}/v_H(1-\beta)(1-\beta\tau) - u_C^2 u_Y^{\beta} v_H^{-2+\beta}(1-\tau)\psi(\xi\psi - \delta_P\sigma); \\ J_{13} &= u_C\left(-\bar{z}/u_Y(1-\beta)(1-\beta\tau) + \xi(1-\tau)\psi + \frac{u_C\beta(1-\tau)\psi(\xi\psi-\delta_P\sigma)}{(1-\beta)\bar{z}}\right); \\ J_{22} &= -\bar{z}(1-\beta) + u_C u_Y/\bar{z}(\xi + \delta_P\sigma)\psi; \\ J_{23} &= -u_Y^{-\beta} v_H^{2-\beta}(1-\beta) - \xi v_H - \frac{u_C v_H \beta(\xi+\delta_P\sigma)\psi}{\bar{z}(1-\beta)}; \\ \bar{z} &= v_H^{1-\beta} u_Y^{1-\beta}; \\ \Upsilon &= (\beta\xi - (1-\beta)\delta_P\sigma). \end{aligned}$$

or $\dot{\mathbf{X}} = \mathbf{J}(\mathbf{X} - \mathbf{X}^*)$, where $\mathbf{J}$ is the Jacobian in (4.7) and J_{ij} are the elements of the Jacobian. To demonstrate the conditions under which the system is stable we use the Routh-Hurwitz theorem.

Using the Routh-Hurwitz theorem, the number of stable roots is equal to the number of variations of sign in the scheme:

$$1 \qquad tr(\bar{J}) \qquad B\bar{J} \equiv \Delta - \det(\bar{J})/tr(\bar{J}) \qquad \det(\bar{J})$$

where $\Delta = J_{11}\,J_{22}\, - J_{12}\,J_{21}\, + J_{22}\,J_{33}\, - J_{32}\,J_{23}\, + J_{11}\,J_{33}\, - J_{13}\,J$.

We now show that a sufficient condition to rule out the case of non-existing stable roots is that $tr(\bar{J}) > 0$ and $\det(\bar{J}) < 0$, noting that if this were to happen we would obtain just one variation in sign independent of the sign of $B\bar{J}$. Thus, the determinant and trace are respectively:

$$\begin{aligned}
\det(\bar{J}) &= -\frac{\xi}{\beta(1+(1-\tau)\psi)}\left((1-\beta)\beta u_C\bar{z}u_Y + \psi\beta u_C^2 u_Y + \psi\delta_P\sigma u_C u_Y^2\left((1-\beta)-u_C/\bar{z}\right)\right): \\
tr(\bar{J}) &= u_C + (\xi u_Y - (1-\beta)\bar{z}) + \frac{u_C u_Y^{\beta} v_H^{-1+\beta}\psi(\xi+(1-\tau)(\xi\psi-\delta_P\sigma))}{1-\beta}.
\end{aligned}$$

It is straightforward to see that sufficient conditions to guarantee saddle-path stability of the steady-states studied in the previous section are the following:

$$\begin{aligned}
\xi u_Y &> (1-\beta)\bar{z} > u_C \\
\xi\psi &> \delta_P\sigma
\end{aligned}$$

whereby, given the calibration values used above, we obtain $0.05u_Y > 0.64\bar{z} > u_C$ and to $\psi > \sigma$. These sufficient conditions are stated for their simplicity; however, we must note that, given our experiments, the steady-state is saddle-path for many parameter combinations that do not respect the sufficient conditions stated above. For instance, we ran an exercise in which we analyzed the eigenvalues of that system from $\sigma = 0$ to $\sigma = 10$, with steps of 0.1 between 0 and 1 and steps of 1 between 1 and 10, for the three cases $\psi = 0.1$, $\psi = 0.5$, and $\psi = 0.9$. We always reached one or two eigenvalues with a negative real part which point out to determinate stability or indeterminate stability. Saddle-path determinate stability always occurs for the low effect of social infrastructure in utility ($\psi = 0.1$) and also occurs for $\psi = 0.5$ and for $\psi = 0.9$ for low values of the effect of social infrastructure on investment. An interesting feature of the situation in which social infrastructure is heavily weighted in utility ($\psi = 0.9$) is that convergence to the steady-state tends to be oscillatory for values of $\sigma > 3$, as complex conjugate values for the stable eigenvalues were found for those combinations of parameters.

5 Simulation

In this section we present the results of a simulation for the model economy when the value of our crucial parameter, σ, is changed.[10] We perform two exercises, one in which σ changes from 0.1 to 0.25 and another in which σ changes from 1 to 1.1. These changes fit in the regions obtained for feasible steady-states and are illustrative exercises. However, we conclude that for several combinations of parameters, the transitional dynamics in this model are very similar. We conclude, in particular, that the transitional dynamics obtained have only minor relevance when compared with steady-state differences in this model. This means that convergence speed is quite high and the economy takes at most 25 years to arrive at the new steady-state. This conclusion supports our complete study of the steady-state properties of the model stated above.

We conclude this section by presenting welfare effects of changes in σ for several combinations of parameters σ and ψ. It is important to look at welfare effects to complete the characterization of the model

[10] We use the Relaxation Algorithm from Trimborn *et al.* (2008).

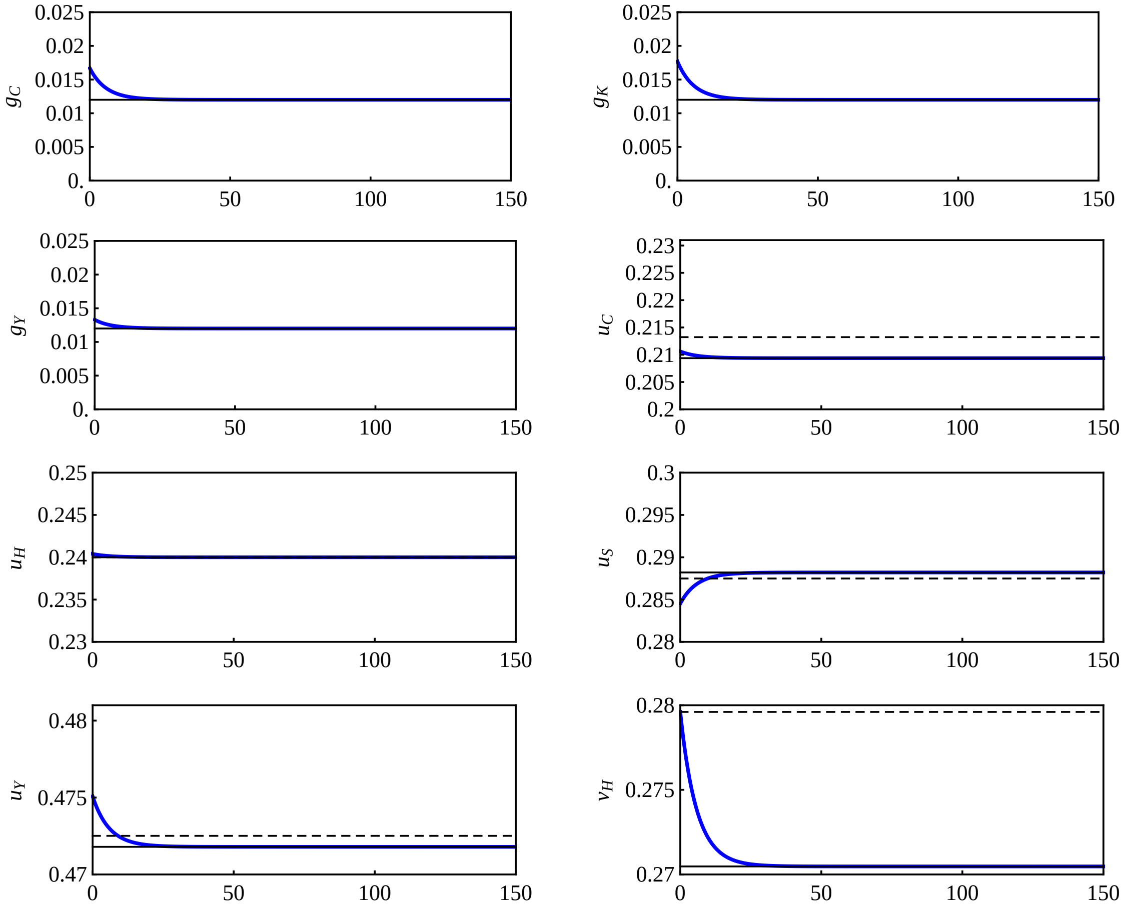

Figure 6: Time paths of representative variables in the model from a steady-state with $\sigma = 0.1$ to a steady-state with $\sigma = 0.25$.
Note: Parameter values are shown at the beginning of the previous section, $\psi = 0.5$. Solid black line refers to the final steady-state and the dashed black line refers to the initial steady-state.

as there is a trade-off between consumption and social infrastructure in this economy. Since an increase in social infrastructure increases utility, it also increases investment. This rise in investment may decrease consumption in the short run. Thus, it is important to measure the relative importance of this short-run negative effect of improving social infrastructure. Figure 6 shows the evolution of the main variables from a steady-state with $\sigma = 0.1$ to a steady-state with $\sigma = 0.25$ for $\psi = 0.5$. In the next figures, we present macroeconomic variables such as growth rates for consumption (g_C), capital (g_{K_P}), and output (g_Y), the shares of human capital allocated to the final good sector, the human capital accumulation sector, and to social capital (u_Y, u_H, and u_S, respectively), and the human capital to physical capital ratio (v_H).

Figure 7 shows the evolution of the main variables from a steady-state in which $\sigma = 1.0$ to a steady-state in which $\sigma = 1.1$ for $\psi = 0.5$.

Once the effect of social infrastructure in preserving physical capital increases, the $v_H = K_H/K_P$ drops, as investment in physical capital begins. This increase in investment is shown in the figure, since g_K increases more than 0.5% in both exercises. The investment growth rate stands above its steady-state level for nearly 10 years. This increase in the growth rate of physical capital is followed by the growth rates for consumption and output. However, the increase of the growth rate of consumption stands below the rise in the physical capital growth rate which is the cause for the drop of the consumption to capital ratio (u_C).

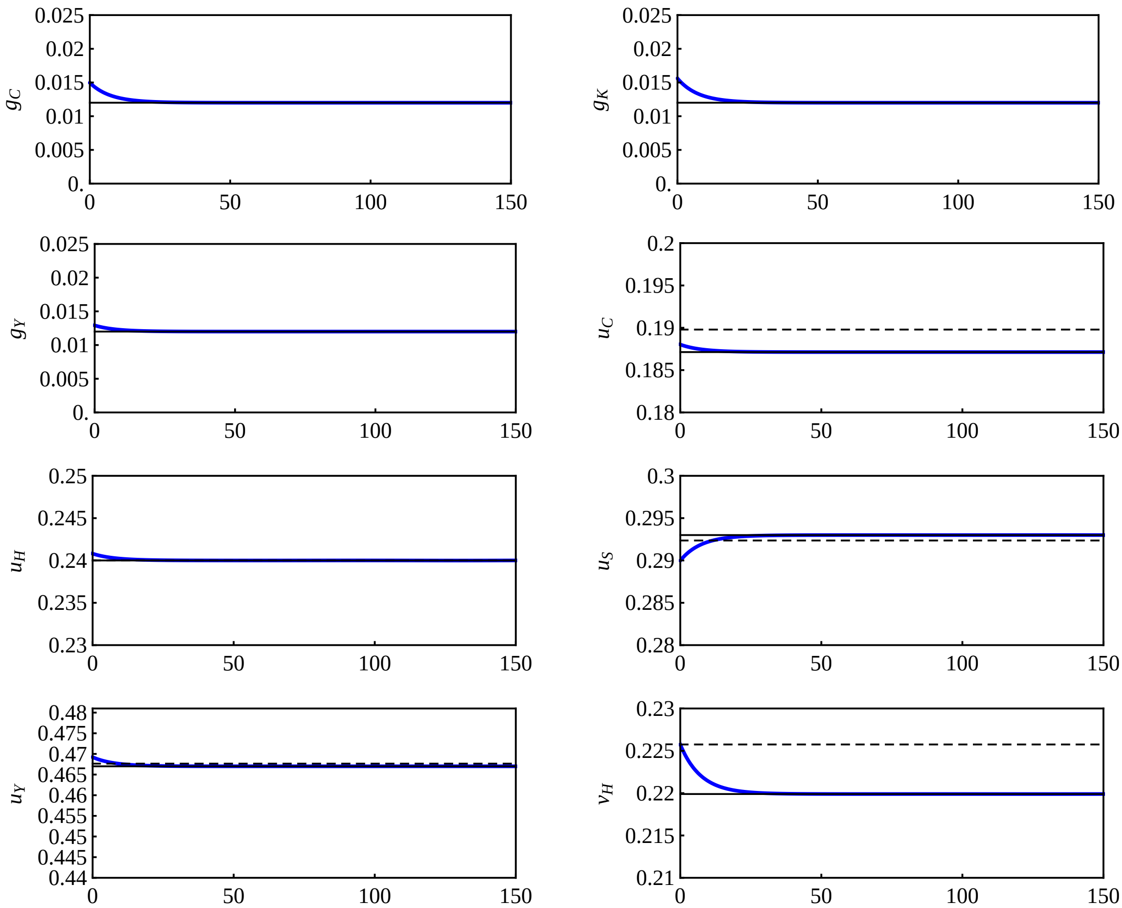

Figure 7: Time paths of representative variables in the model from a steady-state with $\sigma = 1.0$ to a steady-state with $\sigma = 1.1$.

Note: Parameter values are shown at the beginning of the previous section, $\psi = 0.5$. Solid black line refers to the final steady-state and the dashed black line refers to the initial steady-state.

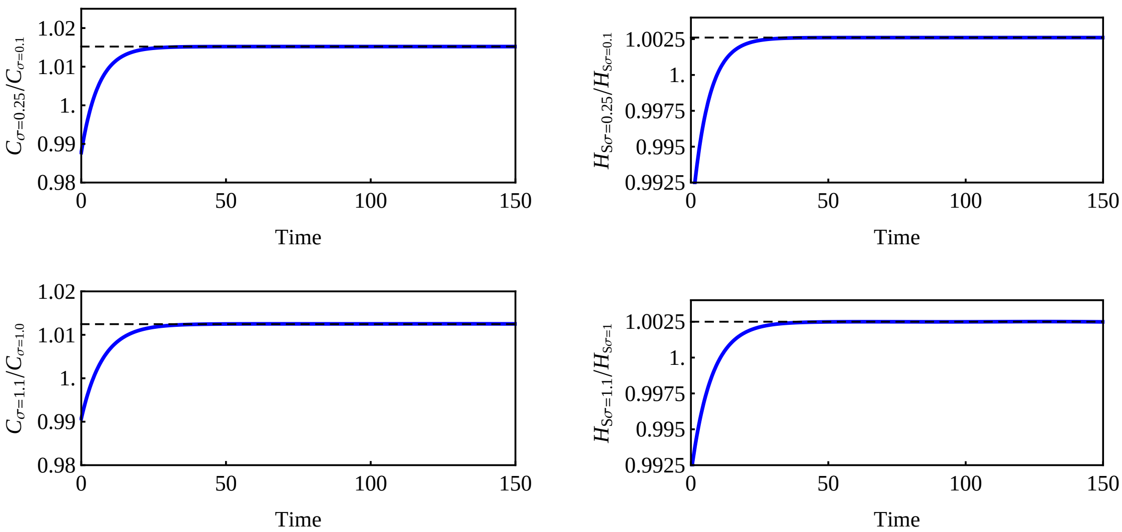

Figure 8: Time paths of consumption and social infrastructure in the model from a steady-state with $\sigma = 0.1$ to a steady-state with $\sigma = 0.25$ and from a steady-state with $\sigma = 1.0$ to a steady-state with $\sigma = 1.1$.
Note: Parameter values are shown at the beginning of the previous section, $\psi = 0.5$. Dashed black lines indicate the final value.

In Figure 6, with a lower effect of social infrastructure in preserving physical capital, the share of human capital allocated to social infrastructure activities first decreases (nearly 0.025%) and then increases to a level that is slightly above the initial steady-state value. This corresponds to an initial increase of human capital allocated to the final good (u_Y), with an almost constant share of human capital in education. This transitional higher allocation of human capital to the production of final good matches the higher investment in physical capital. In Figure 7, however, with a greater effect of social infrastructure in preserving physical capital, we observe a higher drop in v_H, and consequently, a higher effect of increasing protection (due to social infrastructure) in investment.

Transitional dynamic analysis also reveals that the compensation to increase allocation to the final good production due to strengthening of social infrastructure in the economy comes at the expense of allocations to social infrastructure, with minor effects on education.

The intuition behind the transition path for the variables is maintained for exercises in which $\psi = 0.1$ and $\psi = 0.9$.

We also wish to calculate the effect of this rise in σ on welfare. For that we must first calculate a series for consumption C and for the allocation of human capital to social infrastructure activities, H_S, both of which influence utility. Thus this measure takes all of the transitional dynamics into account. Figure 8 shows the evolution of both variables compared with their initial values (by definition, each variable assumes value 1 in the initial steady state).

From Figure 8 we see that there are interesting trade-offs between short and long-run effects that will influence welfare. In both exercises, both consumption and investment in social infrastructure face a short-run negative effect that may be compensated for by a positive effect in the long-run.

Table 1 shows the long-run variations in consumption, in investment in social infrastructure, and in welfare that result from increasing the effect of social infrastructure in protecting investment.

Table 1 - Long-run Effects (%) of Institutional Change in Consumption (C), Social Infrastructure (Hs), and Welfare (W)

	$\sigma = 0.1 \to \sigma = 0.25$	$\sigma = 1.0 \to \sigma = 1.10$
$\psi = 0.1$	$\Delta C = 0.39; \Delta H_S = 0.07; \Delta W = 0.48$	$\Delta C = 0.28; \Delta H_S = 0.05; \Delta W = 0.39$
$\psi = 0.5$	$\Delta C = 1.52; \Delta H_S = 0.26; \Delta W = 1.16$	$\Delta C = 1.25; \Delta H_S = 0.25; \Delta W = 0.79$
$\psi = 0.9$	$\Delta C = 2.26; \Delta H_S = 0.40; \Delta W = 1.58$	$\Delta C = 2.05; \Delta H_S = 0.46; \Delta W = 1.28$

These values indicate a considerable effect on welfare of small variations in the parameter that governs the effect of social infrastructure (σ), effects that oscillate from 0.39% to 1.58%. The welfare effects depend positively and monotonically on the weight of social infrastructure in the utility. Interestingly, the effect on consumption of increasing σ is greater than the effect on social infrastructure (H_S).

6 Conclusion

Following the important literature on institutions and growth, the model in this paper considers that social infrastructure is a specific type of human capital, which allows for preserving physical capital.

Due to the polynomial structure and complexity of the model, we use an innovative methodology in economics, the Gröbner basis, to characterize the feasibility of the steady-state. We conclude that for different regions of the crucial parameters space, two feasible or a unique steady-state could emerge. In particular, unicity is ensured when the effect of social infrastructure in preserving investment is particularly low. When this happens, the steady-state always predicts reasonable values for the shares of human capital allocated to the final good production, education, and social infrastructure. When there are two different steady-states, one is characterized by a higher allocation of human capital to the final good production and high consumption to capital ratio while investing less in social infrastructure, and the other is characterized by lower allocation of human capital to the final good production and consumption and better institutional environment. There is thus a trade-off between present and future determined by allocation of human resources to build social infrastructure. For reasonable intervals of the social infrastructure weight in utility and social infrastructure effect in investment, steady-states are stable, saddle-path or indeterminate, and convergence around the steady-state may be monotonic or oscillatory. Thus, the model that incorporates the role of social infrastructure in preserving physical capital shows a rich set of outcomes.

We also studied transitional dynamics of an economy that strengthens social infrastructure. During the transition path the economy invests more in social infrastructure and allocates less human capital to the final good production, while it induces a phase of higher economic growth.

To summarize, our paper presents an alternative modelling of the effect of social infrastructure on economic growth, through linking social infrastructure with human capital effort which acts on physical capital investment. We conclude for a crucial effect of the quality of social infrastructure (measured by the effect of social infrastructure on investment) on determining if the economy has a unique or two feasible steady-states and whether they are or are not saddle-path stable. Finally, we showed that, for a reasonable calibration set of values for parameters, strengthening the effect of social infrastructure in investment is welfare-improving.

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